Inelastic Capital in Intangible Economies*

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Abstract

Capital accumulation in modern economies is increasingly shaped by intangible investment, which relies heavily on the contributions of specialized workers (e.g., inventors, managers, and entrepreneurs) To examine the macroeconomic implications of the growing importance of intangible investment, we develop and calibrate a general neoclassical model where capital formation requires a mix of investment goods (tangible investments) and specialized labor (intangible investments). We show that rising intangibles makes the supply of capital more inelastic owing to the limited supply of specialized labor. Rising intangibles also change the incidence of capital taxation: whereas the tax burden falls entirely on production workers in the neoclassical growth model, it is borne mostly by specialized workers and capital owners in intangible economies.

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1 Introduction

Capital formation increasingly takes the form of intangible investments, in particular research and development (Corrado and Hulten, 2010). One key property of intangible investment, relative to tangible investment, is that it critically relies on talented, specialized workers (e.g., managers, inventors, entrepreneurs etc). This paper examines the implication of this shift towards intangible capital on the economy.

Our main result is that intangible economies exhibit a low capital supply elasticity owing to the fact that specialized labor is in finite supply. In other words, aggregate investment in intangible economies is *less* responsive to demand shocks in the capital market (e.g., interest rates, tax rates, productivity). The reason is that, as aggregate investment increases, the economy runs out of specialized labor, leading to a rise in their wage and thus an increase in the marginal cost of investment.

Overview of the paper. The starting point of the paper is that investment goods (i.e., machines, computers, plants) must be paired with specialized labor (i.e., inventors, entrepreneurs, financiers) and accumulated capital (i.e., building on past research, adjustment frictions) in order to produce business capital. To capture this idea, in Section 2 we write down a standard neoclassical model of capital accumulation with a general aggregate investment function:

$$\text{capital formation} = F\left(\text{investment goods, existing capital, investment labor}\right). \tag{1.1}$$

In the baseline, we parametrize (1.1) using a constant return to scale Cobb-Douglas function and close the model with inelastic labor supply (i.e., there is a fixed number of investment workers). Our general specification nests and generalizes several important models used in the literature. For instance, the neoclassical growth model (NGM) only requires investment good, the Q-theory of investment requires goods and existing capital, while models of "sweat capital" require both existing capital and labor.

We then characterize the capital supply elasticity, which governs how responsive firms are to capital demand shocks.[1] In the NGM, capital supply is infinitely elastic, since the final good can be frictionlessly transformed into capital. In that case, small demand-side shocks (e.g., interest rate) have large and immediate effect on capital accumulation. In contrast, when investment requires labor in fixed supply, the supply elasticity is finite, even in the long-run. We show that a higher degree of intangibility in capital formation (i.e., Cobb-Douglas coefficient on investment labor to goods in 1.1)

In Section 3, we map the firm budget constraint in the model to the data. We construct a dataset on the uses of capital income in the US nonfinancial corporate sector, based around the identity:

$$\text{capital income} = \text{capital payout} + \text{tangible investments} + \text{intangible investments}. \tag{1.2}$$

We focus on public firms (Compustat-CRSP) over the 1972-2022 period, where we have full financial statements, which include noncash payments (e.g., stock options, acquisitions financed with stocks).

[1] To be precise, the supply elasticity of capital in the model is the elasticity of the capital stock (at some horizon) with respect to a change in the value of capital (i.e., the marginal q).

Building on the existing literature, we make three key adjustments to the accounting data. First, we treat nonproduction labor costs as intangible investment, rather than production costs (see, e.g., Corrado, Hulten and Sichel, 2005; Corrado, Hulten and Sichel, 2009; Eisfeldt and Papanikolaou, 2013; Peters and Taylor, 2017;Koh, Santaeulàlia-Llopis and Zheng, 2020). The idea is that if an employee dose not contribute to the production process, then they must contribute to capital formation, broadly defined. Second, we account for IPOs and acquisitions net of de-listings and allocate the implied cashflows to tangible and intangible investments. Third, we account for noncash payments, treating them as economically equivalent to cash payments. This is particularly important for nonproduction labor (inventors, managers) who earn a large share of their compensation in stocks (see, e.g., Eisfeldt, Falato and Xiaolan, 2023).

We obtain an annual dataset on the aggregate (and industry-level) uses of capital income, which maps exactly into the firm budget constraint (1.2) in the model. Note that our accounting is fully consistent with national accounting, which now acknowledges that some form of labor expenses are indeed investment. However, we go further than the BEA and consider any nonproduction labor expenses as investment. Corrado, Haskel, Jona-Lasinio and Iommi (2022) discuss this topic in detail, highlighting the fact that the following expenses are not currently considered investment in the national accounts: "market research and branding, operating models, platforms, supply chains, distribution networks, employer-provided training, attributed designs (industrial), and financial product development".

Finally, in Section 4, we calibrate the key elasticities of the model using data on the uses of capital income and valuations (intangible share, total investment yield, and aggregate capital share), as well as a short-run tax elasticity of investment taken from Chodorow-Reich, Smith, Zidar and Zwick (2024). Using the calibrated model we simulate a large (25 pp.) rise in the share of intangible investment, consistent with our evidence from the US corporate sector over 1972-2022. We find that a shift towards intangibles of the magnitude that we have seen in the data implies a significant redistribution of aggregate income away from production labor towards investment labor (-6 pp. of GDP). The model also predicts that the economy becomes more inelastic: the supply elasticity declines by roughly half in the long-run and one-third in the short-run.

To understand the economic implications of such a shift, we quantify the incidence of capital taxes in our economy and in benchmark models. In the neoclassical growth model (henceforth NGM), the capital supply elasticity is infinite. As a result, capital tax cuts have a large effect on investment, and end up being born by production workers via higher wages. In intangible economies, however, tax cut have a weaker effect on investment, hence a lower effect on production worker wages. As a result, capitalists and investment labor absorb more of the shock via revaluation gains (for capitalists) and higher wages (for investment labor). In the calibrated model, the incidence of capital taxes is roughly half for capitalists, a quarter for production labor, and a quarter for investment labor.

Literature review. A growing literature documents a rise in intangible capital (see, e.g., Eisfeldt and Papanikolaou, 2013, Peters and Taylor, 2017, Eisfeldt, Falato and Xiaolan, 2023, and Corrado, Haskel, Jona-Lasinio and Iommi, 2022). Beyond being harder to measure, intangible capital may exhibit distinct economic properties relative to tangible capital. In Eisfeldt and Papanikolaou (2013), intangible capital is unique because it is embedded in key talents. In Crouzet, Eberly, Eisfeldt and Papanikolaou (2022), intangible capital tends to be non-rival—allowing it to be used simultaneously in different production streams—and is characterized by limited excludability, which prevents firms from capturing all the associated benefits or rents. In this paper, we shift our focus away from the economic properties of intangible capital to its distinct inputs, namely, that it is created by specialized labor in finite supply.

The closest study to ours is Luttmer (2018), which studies an economy in which households supply both managerial and production labor—with managerial labor contributing to both production and investment. Like our paper, Luttmer (2018) highlights that organizational capital is produced using a specialized input by discussing the implications of a fixed supply of managerial capital. Another closely related paper is Bhandari and McGrattan (2021), which highlights that a significant portion of small firms' value stems from organizational capital accumulated through the owner's effort. The paper further explores the implications of this finding for the taxation of non-corporate businesses. Relative to these papers, we provide new data and moments to calibrate a flexible theory of capital supply.

Finally, our paper offers an alternative explanation for investment stagnation despite high Tobin's Q inferred from the data. The existing literature focuses on markups and market power (e.g., Gutiérrez and Philippon, 2017; Crouzet and Eberly, 2019; Barkai, 2020; Ball and Mankiw, 2023; De Ridder, 2024). In contrast, we emphasize the fact that capital formation requires specialized labor (i.e., entrepreneurs, inventors, etc.) which constrains the extent to which investment goods can be transformed into productive capital. Put differently, in our paper, the "fixed factor" that limits the ability (or willingness) of firms to scale is not the fact that it would lower their price, but, rather, that it would increase their labor cost.

2 Stylized model

2.1 Setup

Output production. We focus on a representative firm. Output Y_t (i.e., the final good) is produced using capital K_t and production labor $L_{Y,t}$ through the standard Cobb-Douglas production function

$$Y_t = z_{Y,t} K_t^{\alpha} L_{Y,t}^{1-\alpha}, \qquad \text{(output production)}$$

where $z_{Y,t}$ represent Hicks-neutral productivity in production (TFP).

Capital formation. Our key departure from the standard neoclassical growth model is that the final good cannot be frictionlessly transformed into productive capital. Consistent with evidence on capital formation in the modern corporate sector, we assume that producing and installing new productive units of capital also requires specialized labor (i.e., inventors, entrepreneurs, financiers) as well as accumulated capital (i.e., building on past research, adjustment frictions). We represent this with a standard capital accumulation equation and a general capital formation function:

$$K_{t+1} = (1-\delta)K_t + H_t,$$
$$\text{where} \quad H_t \equiv z_{H,t} K_t^{\theta} \left(I_t^{\chi} L_{H,t}^{1-\chi} \right)^{1-\theta}. \qquad \text{(capital formation)}$$

First, note that capital formation requires both output I (tangible investment) and specialized labor L_H (intangible investment); the parameter $\chi \in (0,1)$ governs the tangibility of capital. Second, there are diminishing marginal returns in both tangible inputs and specialized labor (holding constant the existing stock of capital); the capital share in investment is governed by $\theta \in (0,1)$. Finally, $z_{H,t}$ represent Hicks-neutral productivity in capital production (IST shock).[2]

Our specification of the investment function nests and generalizes several important models used in the literature. Our model nests the neoclassical growth model (NGM) in the special case where investment only requires goods ($\theta = 0$, $\chi = 1$). In this case the capital formation equation simplifies to $H_t = z_{H,t} I_t$, which implies that goods and capital can be transformed into each other using a (potentially time-varying) linear technology, as in Greenwood et al. (1997).

Another important special case is where investment requires both goods and capital, but not specialized labor ($\chi = 1$). In this case, the capital accumulation becomes $H_t = z_{H,t} K_t^{\theta} I_t^{1-\theta}$. This type of investment function is equivalent to the traditional q-theory of investment (Uzawa, 1969, Hayashi, 1982) which models capital accumulation as $K_{t+1} = (1-\delta) + K_t \phi(I_t/K_t)$, where $\phi(\cdot)$ is a concave function.[3] Hence, our theory corresponds to the case in which $\phi(I_t/K_t) = (I_t/K_t)^{1-\theta}$. The key insight is that the capital share θ is directly related to the curvature of the adjustment cost function in the traditional q-theory of investment. This parameter governs the *short-run* fluctuations in investment, as, in this particular formulation , the elasticity of today's investment I_t to today's q_t is $1/\theta$.

Another important special case is where investment requires both capital and specialized labor, but not goods themselves ($\chi = 0$). In this case, the capital accumulation becomes $H_t = z_{H,t} K_t^{\theta} L_{H,t}^{1-\theta}$. This corresponds to models of "sweat capital", where firm expansion requires organizational capital (e.g., Luttmer, 2011; Bhandari and McGrattan, 2021). A similar idea is at the core of several important models, such as the Melitz (2003) model (firm entry requires labor) or the Romer (1986) model (innovation requires labor).

Firm optimization. The representative firm takes as given the interest rate r_t, as well as wage rates for production and investment labor $(w_{Y,t}, w_{H,t})$. It chooses production labor $L_{Y,t}$, investment labor $L_{H,t}$,

[2]Because we choose a Cobb-Douglas production function, this can alternatively represent technology shocks that make it easier to install capital, shocks that make investment good more productive (as in Greenwood et al., 1997), or shocks that make investment labor more productive.

[3]After Hayashi (1982), the q-theory of investment has evolved away from modeling capital formation as concave in I/K (like we do) towards cost functions that are convex in I/K. The key difference is that adjustment costs appear in the budget constraint while concave capital formation appear in the capital accumulation equation. While our model does not exactly next the quadratic adjustment cost case, it shares the key insight that there are short-run decreasing returns to scale in capital accumulation.

and tangible investment I_t to maximize the present value of future payouts:

$$V_0 = \max_{\{L_{Y,t},L_{H,t},I_t,K_{t+1}\}} \sum_{t=1}^{\infty} R_{0\to t}^{-1} D_t,$$

$$\text{s.t.} \qquad D_t = Y_t - I_t - w_{Y,t}L_{Y,t} - w_{H,t}L_{H,t}, \qquad \text{(firm budget constraint)}$$

where $R_{0\to t} = \prod_{s=0}^{t}(1+r_s)$ denotes the cumulative return of one dollar invested between 0 and t. This maximization is subject to the equations for production, capital formation, and capital accumulation above. The corresponding Lagrangian is

$$\mathcal{L}_0 = \sum_{t\geq 1} R_{0\to t}^{-1}\Big(Y_t - I_t - w_{Y,t}L_{Y,t} - w_{H,t}L_{H,t}\Big) + \sum_{t\geq 1} R_{0\to t}^{-1} q_t \Big((1-\delta)K_t + H_t - K_{t+1}\Big).$$

The Lagrange multiplier q_t can be interpreted as the shadow price for productive capital units. Solving the representative firm problem yields the following set of optimality conditions:

$$w_{Y,t} = (1-\alpha)\frac{Y_t}{L_{Y,t}}, \qquad \text{(firm foc } L_{Y,t}\text{)}$$

$$w_{H,t} = (1-\theta)(1-\chi)\frac{q_t H_t}{L_{H,t}}, \qquad \text{(firm foc } L_{H,t}\text{)}$$

$$1 = (1-\theta)\chi\frac{q_t H_t}{I_t}, \qquad \text{(firm foc } I_t\text{)}$$

$$R_{t+1}q_t = \alpha\frac{Y_{t+1}}{K_{t+1}} + q_{t+1}\left((1-\delta) + \theta\frac{H_{t+1}}{K_{t+1}}\right), \qquad \text{(firm foc } K_{t+1}\text{)}$$

with the transversality conditions $\lim_{T\to\infty} R_{0\to t}^{-1} q_T K_{T+1} = 0$.

Household optimization. Since the focus of the paper is on the firm side, we only provide a stylized microfoundation of the household problem. The representative household takes as given the sequence of prices $\{w_{Y,t}, w_{H,t}, R_t\}_{t=1}^{\infty}$ and chooses a sequence of consumption and wealth $\{C_t, V_t\}_{t=1}^{\infty}$ to maximize welfare:

$$U_0 = \max_{\{C_t,V_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}. \qquad (2.1)$$

$$\text{s.t.} \quad C_t + V_t = w_{Y,t}L_{Y,t} + w_{H,t}L_{H,t} + R_t V_{t-1},$$

where V_0, the initial level of wealth, is given. For simplicity, we assume in this section that the representative household supplies a fixed quantity of production labor $L_{Y,t} = 1-\mu$ and investment labor $L_{H,t} = \mu$; we will consider the more general case in which labor is elastically supplied in Section 4, when taking the model to the data. The household optimality with respect to consumption gives the usual Euler equation:

$$C_t^{-\gamma} = \beta R_{t+1} C_{t+1}^{-\gamma}. \qquad \text{(worker foc } C_t\text{)}$$

Equilibrium. An equilibrium is an initial condition K_0, an allocation $\{L_{Y,t}, L_{H,t}, I_t, K_t\}_{t\geq 1}$, and prices $\{w_{H,t}, w_{L,t}, R_t, q_t\}_{t\geq 1}$ that solve the firm problem (firm foc $L_{Y,t}$, firm foc $L_{H,t}$, firm foc I_t, firm foc K_{t+1})

and household problem (worker foc C_t), with $L_{Y,t} = 1 - \mu$ and $L_{H,t} = \mu$.

2.2 Steady-state

We now characterize the steady-state of the model, assuming that the productivity of the output and capital production functions remain fixed over time $z_{Y,t} = z_Y$ and $z_{H,t} = z_H$. We consider the more general case of a balanced growth path where the productivity of investment and production labor grow at some exogenous rate in Section 4.

Equilibrium in the capital market. It is useful to think of the steady-state (or long-run) quantity of capital as determined by an equilibrium between the demand and supply of capital. The demand for capital directly obtains by combining firm foc K_{t+1} with the steady state condition $H = \delta K$:

$$z_Y \alpha K^{-(1-\alpha)} (1-\mu)^{1-\alpha} = q\left(r + (1-\theta)\delta\right) \qquad \text{(capital demand)}$$

This equation can be seen as a market pricing equation, which pins down the price of capital q given the flow of payments to capital holders and the interest rate r. Another, equivalent, interpretation of (capital demand) is that firms demand capital until the marginal productivity of capital (the left-hand side) coincides with its user cost (the right-hand side), as in Hall and Jorgenson (1967).[4] This equation traces a downward slopping demand for capital: a higher capital price q increases its user cost (since a firm would need to borrow more to purchase one unit of capital), reducing the quantity of capital demanded by firms.

The supply of capital obtains by plugging optimal investment (firm foc I_t) into the steady-state version of (capital formation):

$$0 = \underbrace{z_H K^{\theta} \Big(\left((1-\theta)\chi q \delta K\right)^{\chi} \mu^{1-\chi} \Big)^{1-\theta}}_{=H} - \delta K \qquad \text{(capital supply)}$$

A higher shadow value of capital q pushes firms to invest more (firm foc I_t), which increases the steady-state supply of capital (capital formation). In the particular case where investment is purely tangible ($\chi = 1$), the supply of capital is perfectly elastic and this equation pins down q. When capital formation requires investment labor through the form of intangible investment ($\chi < 1$), however, this equation traces an upward slopping supply curve for capital.

The combination of (capital supply) and (capital demand) pins down the equilibrium price and quantity of capital in steady-state. The equilibrium is determined by the intersection between these two curves.

Lemma 2.1. *The steady-state of the model is given by the intersection of the following capital demand and supply*

[4] The analogy provided by Hall-Jorgenson is that "the firm may be treated as accumulating assets in order to supply capital services to itself".

curves:

$$K = (1-\mu)\left(\frac{r+(1-\theta)\delta}{z_Y\alpha}q\right)^{-\frac{1}{1-\alpha}} \qquad \text{(capital demand}')$$

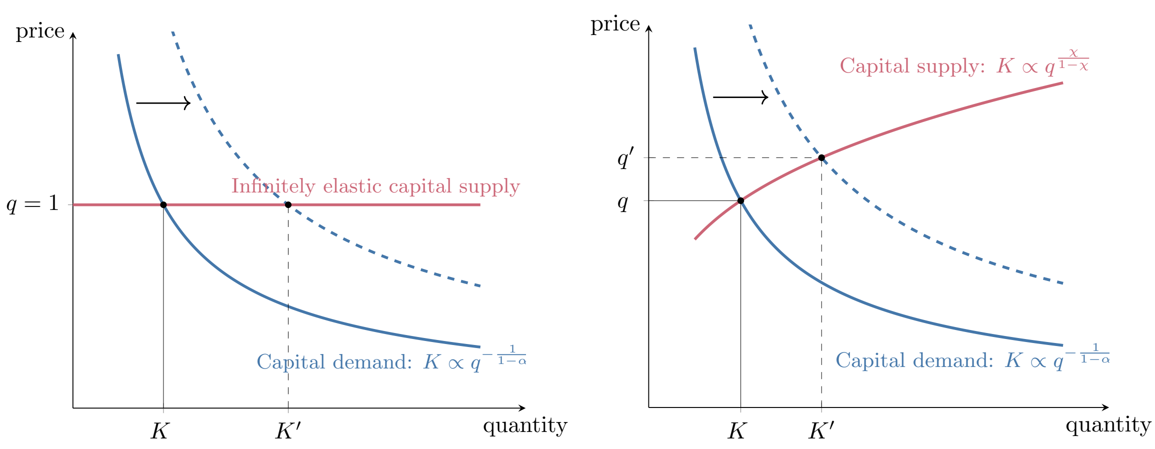
$$K = \mu\left(\left(\frac{z_H}{\delta}\right)^{\frac{1}{\chi(1-\theta)}}(1-\theta)\chi\delta q\right)^{\frac{\chi}{1-\chi}} \qquad \text{(capital supply}')$$

where $R = 1/\beta$ is pinned down by (worker foc C_t)

On the one hand, a higher capital price q increases the user cost of capital, decreasing the firm's demand for capital (capital demand′). On the other hand, a higher capital price q increases firm investment, which decreases the steady-state of capital (capital supply′). The (long-run) elasticity of capital demand to q is $-1/(1-\alpha)$ while the (long-run) elasticity of capital supply to q is $\chi/(1-\chi)$. This elasticity spans values from zero (when investment is fully intangible) to infinity (when capital is fully tangible, as in the neoclassical growth model). Notice that the long-run capital supply elasticity does not depend on θ; as we discuss below, the parameter θ only determines the short-run capital supply elasticity.

Long-run effects of shifts in the demand for capital. The fact that, in the presence of intangibles, the supply of capital is inelastic has key implications for the long-run effect of a shift in the demand of capital — say, due to a permanent increase in the household's time preference parameter β or a permanent increase in output productivity z_Y.

To visualize this fact, Figure 1 plots the steady-state demand and supply for capital. Panel (a) corresponds to the neoclassical growth model ($\chi = 1$), while Panel (b) corresponds to an intangible economy ($\chi < 1$ in Panel b). In the neoclassical growth model (Panel a), the supply of capital is perfectly elastic, so an outward shift in capital demand results in a higher steady-state capital stock with no change in the rental rate of capital. In contrast, in a model with intangibles (Panel b), the supply of capital is inelastic, so a shift in the demand for capital is only partially absorbed by a rise in capital, and part of the adjustment takes the form of a higher capital price (i.e., a higher firm value relative to its capital stock).



(a) Neoclassical growth model ($\chi = 1$) (b) Model with intangibles ($\chi < 1$)

Figure 1: Long-run equilibrium in the capital market

Short-run effects of shifts in the demand for capital. Capital is also inelastic in the short-run, due to the joint presence of specialized labor ($\chi < 1$) and existing capital ($\theta > 0$) in the capital production function. To examine the short-run supply elasticity of capital, consider a perturbation $\{\mathrm{d}\log q_t\}_{t=1}^{\infty}$. As shown in Appendix A.2, the response of K_{t+s} at any horizon $s \geq 0$ is given by:

$$\mathrm{d}\log K_{T+1} = \frac{\chi}{1-\chi} \cdot \delta \frac{\chi(1-\theta)}{1-\chi(1-\theta)} \sum_{s=0}^{T} \left(1 - \delta \frac{\chi(1-\theta)}{1-\chi(1-\theta)}\right)^{s} \mathrm{d}\log q_{T-s}$$

(Short-run capital supply elasticity)

While the capital share of investment θ did not matter for the long-run capital elasticity, it is the key determinant of the short-run capital elasticity. Capital being an input in production $\theta > 0$ makes capital less elastic in the short-run. This is exactly the logic behind the q-theory of investment, which emphasizes the presence of capital adjustment costs in the short-run. As in the long-run, capital supply is more elastic when tangibility χ is high. Finally, note that we recover the long-run elasticity as the limiting case of the short-run elasticity after setting $\mathrm{d}\log q_t = \mathrm{d}\log q$ for all t and taking the limit $t \to \infty$, which yields, as in Lemma 2.1, $\mathrm{d}\log K = \frac{\chi}{1-\chi}\,\mathrm{d}\log q$.

Table 1: Comparing the supply elasticity of capital across benchmark models

Model	Constraint		Elasticity of capital	
	θ	χ	short-run $\left(\frac{\partial \log K_{t+1}}{\partial \log q_t}\right)$	long-run $\left(\frac{\partial \log K}{\partial \log q}\right)$
Neoclassical growth	$\theta = 0$	$\chi = 1$	$+\infty$	$+\infty$
Q-theory	–	$\chi = 1$	$\delta \frac{1-\theta}{\theta}$	$+\infty$
Sweat capital	–	$\chi = 0$	0	0
Our model	–	–	$\delta \frac{\chi(1-\theta)}{1-\chi(1-\theta)}$	$\frac{\chi}{1-\chi}$

Notes. θ the investment capital share; χ is the tangibility of investment; δ is the depreciation rate.

Table 1 compares the short- and long-run supply elasticity of capital in our model relative to different benchmark models. There are two key takeaways. First, in fully-tangible economies ("NGM" and "q-Theory"), capital is infinitely elastic in the long-run. This is because the relative price of capital in terms of goods must be one since they are the same thing. Adding a positive capital share of investment ($\theta > 0$) makes the short-run elasticity finite (see "q-theory"), but does not solve the problem of infinite elasticity in the long-run. Second, intangibility (i.e., $\chi < 0$) makes the supply elasticity of capital finite, both in the short- and long-run (see "sweat capital"). This is because labor supply is finite (zero in our case), which makes the response of capital formation to q constrained by the fixed pool of investment worker. In other words, investment booms in intangible economies raise the wages of investment labor, hence dampening the size of the boom.

2.3 Wage gap between investment and production workers

Capital supply is inelastic in our model because capital formation requires an input that is fixed at the aggregate level: investment labor. In equilibrium, after a shift in the demand for capital, firms respond by increasing investment, which bids up the wages of investment workers who are in limited supply. Hence, the flip-side of inelastic capital is that shifts in the demand for capital will affect the wage gap between investment and production workers.

To see this, one can combine (firm foc $L_{Y,t}$) and (firm foc $L_{H,t}$) to express the ratio of payments between investment and production workers:

$$\frac{w_H L_H}{w_Y L_Y} = \frac{(1-\theta)(1-\chi)}{1-\alpha}\frac{q_t H_t}{Y_t} = \frac{(1-\theta)(1-\chi)}{1-\alpha}\frac{\delta\alpha}{r+\delta(1-\theta)} \tag{2.2}$$

This first equality reflects that, due to our Cobb-Douglas assumptions for our production functions, investment and production workers receive a fixed share of the revenues from their respective sectors. The second equality uses (capital demand), which pins down the ratio between output and the value of the capital stock in steady state.

Therefore, shocks that disproportionately affect the investment sector will increase the wage gap w_H/w_Y. Consider, for instance, an increase in the household's time preference β generating a decline in r. As shown in (2.2), the decline in r increases the steady-state value of capital relative to output, which increases the wage gap between investment and production workers. Similar, a rise in intangibles (a decrease in χ) increases the wage gap between the two type of workers.

Put differently, while all workers are paid their marginal products, investment workers produce cash-flows that have a longer duration than production workers. As a result, as interest rates decline, the marginal product of investment workers increases relative to production workers, and so the wage gap between the two workers increases as a result.

Our theory for the wage gap between investment and production workers provides an alternative view of the "skill-biased" view of technical change (see, for instance, Krusell et al., 2000). In our model, investment workers benefit from investment booms because they are key inputs in capital formation, not because they are complements with capital in production. One key difference between the two theories is that, in our model, any shock that make capital more valuable increases the wage of investment workers — whether or not this higher capital value ultimately translates into a higher capital stock. We will return to this idea when quantifying the effect of the rise in intangibility on the wage gap between investment and production workers in Section 4.

3 Measuring tangible and intangible investment

We now use data on the U.S. corporate sector to calibrate our model, and in particular the new parameter χ. To do so, we focus on measuring the uses of capital income in the data.

3.1 Model-implied distribution of capital income

We now describe the distribution of income in our economy. GDP in our economy should be defined as $Y_t + w_{H,t} L_{H,t}$, not Y_t. This captures the fact that the economy produces a quantity Y_t of goods (used for consumption or for tangible investment) as well as a quantity $w_{H,t} L_{H,t}$ of intangible investment. While this logic is fully consistent with the *System of National Accounts*, distinguishing between payments to production versus investment labor is difficult in practice and the BEA's methodology is known to have limitations in that regard (Corrado et al., 2022).

Consistently with national accounts, we then define capital income as gross output net of production costs, which gives $\Pi_t \equiv Y_t - w_{Y,t} L_{Y,t}$.[5] Plugging this equality into (firm budget constraint), we obtain the following accounting identity, which describes the *uses* of capital income (i.e., how do firms spend their profits):

$$\underbrace{\Pi_t}_{\text{capital income}} = \underbrace{I_t}_{\text{tangible investments}} + \underbrace{w_{H,t} L_{H,t}}_{\text{intangible investments}} + \underbrace{D_t}_{\text{payouts}}. \tag{3.2}$$

It says that capital income has three uses: tangible investments, intangible investments, and payments to owners of the firm (henceforth "payouts").

Manipulating the first-order conditions of the firm implies that, in the long run, the distribution of capital income in the economy is given by:[6]

$$\frac{I}{\Pi} = \chi \frac{(1-\theta)\delta}{r + (1-\theta)\delta}; \qquad \frac{w_H L_H}{\Pi} = (1-\chi) \frac{(1-\theta)\delta}{r + (1-\theta)\delta}; \qquad \frac{D}{\Pi} = \frac{r}{r + (1-\theta)\delta}.$$

This equation expresses the share of tangible investment, the share of intangible investment, and the payout share in steady state in terms of only four parameters. For the payout share, what matters is the discount rate r and the dilution rate $\delta(1-\theta)$, which itself depends on depreciation rate δ and the capital share of investment θ. We use the term "dilution rate" for $\delta(1-\theta)$ because it corresponds to the extent to which a capitalist that consumes the entirety of capital income in a given period gets diluted over time.

The lower the discount rate r, the lower the share of capital income paid out the capitalists. Given a discount rate, a higher dilution rate $\delta(1-\theta)$ implies a lower current payout. This is because, in high dilution economies, capitalists must receive a higher current payout to guarantee a given long-run return (the payout stream has a low duration). As discussed earlier, the capital share of investment θ is key to determining how much capitalists get diluted, and hence the duration of their wealth.[7] What is

[5] In our economy, the standard GDP identity (i.e., income equals expenditures) can then be written as:

$$\underbrace{\overbrace{w_{Y,t} L_{Y,t} + w_{H,t} L_{H,t}}^{\text{labor income}} + \overbrace{\Pi_t}^{\text{capital income}}}_{\text{income}} = \underbrace{\overbrace{Y_t - I_t}^{\text{consumption}} + \overbrace{I_t + w_{H,t} L_{H,t}}^{\text{investment}}}_{\text{expenditures}}. \tag{3.1}$$

[6] Evaluating (firm foc I_t) and (firm foc $L_{H,t}$) in steady-state, we have

$$I = \chi(1-\theta) q H = \chi \frac{(1-\theta)\delta}{r + (1-\theta)\delta} \Pi; \qquad w_H L_H = (1-\chi) \frac{(1-\theta)\delta}{r + (1-\theta)\delta} \Pi,$$

[7] The duration of a payout stream $\{D_t\}_{t=0}^{\infty}$ is defined as the value-weighted time to maturity $\sum_{t=1}^{\infty} \frac{R_{0\to t}^{-1} D_t}{\sum_{s=1}^{\infty} R^{-s} D_s} \cdot t$ and is typically used to measure the interest-rate sensitivity of an asset price. In the model, the steady-state duration of payouts is equal to

not paid out is invested, in proportion to the tangibility of investment parameter χ.

3.2 Data

We use Compustat-CRSP merged data covering the period from 1972–2022. We use usual screens to focus on the nonfinancial corporate sector. Our goal is to construct industry-level series for: capital income Π, tangible investments I, intangible investments $w_H L_H$, and payments to capitalists D.

To measure the use of capital income in the data, we start from the statement of cashflows, as reported in Compustat. The key accounting identity is:

$$\text{cf from operations} + \text{cf from financing} + \text{cf from investing} = 0 \tag{3.3}$$

The first term is the cashflows from operations, the second is the cashflows from financing activities, and the third is the cashflow for investing activities. We use the convention that changes in cash balances represent net payments to debt holders. Furthermore, we will later use the fact that cashflows from financing and investing can be decomposed into:

$$\text{cf from financing} = \text{cf from financing (equity)} + \text{cf from financing (debt)} \tag{3.4}$$

$$\text{cf from investing} = \text{cf from investing (capex)} + \text{cf from investing (acquisition)} \tag{3.5}$$

Equation (3.3) allows us to account for 100% of the cash that comes in due to profits (cashflows from operations), the net cash that comes out to pay owners (cashflows from financing activities), and the cash that comes out due to investment (cashflows from investment activities).

Notice that the cashflow identity is the financial accounting counterpart of the firm budget constraint (3.2) in the model. The mapping between model concepts and financial accounting terminology is summarized below:

$$\underbrace{\Pi_t}_{\substack{\text{capital income =}\\ \text{cashflows from operations}}} = \underbrace{(\Pi_t - w_{H,t}L_{H,t} - I_t)}_{\substack{\text{payments to capitalists =}\\ -\text{cashflows from financing}}} + \underbrace{(w_{H,t}L_{H,t} + I_t)}_{\substack{\text{total investment =}\\ -\text{cashflows from investment}}} \tag{3.6}$$

Building on the existing literature, we conduct some data imputations and adjustments, which we describe below.

Adjustment #1: expensed payments to investment labor. One issue with accounting data is that the distinction between an expense (which are subtracted from sales to obtain cashflows from operation) and investment (which are not) can be arbitrary, especially in the case of intangibles.

The existing literature shows that this issue leads to an understatement of cashflows from operations and a corresponding understatement of (minus) cashflows from investment.[8] We follow the existing literature and correct cashflows from operation by adding 30% of SG&A expenses and 100% of R&D

$\frac{1}{r+(1-\theta)\delta}$. Hence, a higher dilution $(1-\theta)\delta$ implies lower duration.

[8]See Peters and Taylor (2017) for evidence from firm-level data and Koh et al. (2020) from aggregate data.

expenses. This is meant to account for the incorrect expensing of investment labor expenses, which instead should be treated as a capital expenditure.

$$\text{cf adjustment (investment labor)} \equiv 0.3 \cdot \text{SG\&A} + \text{R\&D}. \tag{3.7}$$

Adjustment #2: net entry in sample. When a private firm becomes public or goes public, it effectively represents a negative cashflow for passive capitalists, who re-balance their portfolio to always own the market. Similarly, when a firm exits or gets acquired, it represents a positive cashflow. We now describe how we measure the contribution of net firm entry to market capitalization growth.

First, we introduce a decomposition of the total market value growth (at the aggregate or within an industry) between time t and $t+1$ as the sum of three components due to stayers, entrants, and exits. Formally, we have that

$$g_{\text{total}} = g_{\text{stayer}} + g_{\text{entry}} + g_{\text{exit}}, \tag{3.8}$$

where the components are defined in this footnote.[9]

Using these definitions, we define the net cashflow due to net firm entry as

$$\text{cf adjustment (net entry)} \equiv -(g_{\text{entry}} + g_{\text{exit}}) \cdot \text{market capitalization}. \tag{3.9}$$

It corresponds to the net cash that a passive capitalists who owns the market receives due to net firm entry during a period.

Adjustment #3: payments in stocks. How should we record a payment when it is the form of stocks? First, we can decompose the growth in market value of a stayer into growth of price per share and the growth in number of shares. To match the timing of the payment to the timing of the stock issuance, we use the fully-diluted share count (which includes outstanding shares, as well as all possible sources of potential shares such as stock options and reserved shares).

Formally, we can decompose the growth of the market capitalization of stayers (i.e., firms who do not exit the sample) defined in (3.8) into a price and number of shares component:

$$g_{\text{stayer}} = g_{\text{price}} + g_{\text{shares}}, \tag{3.10}$$

where the components are defined in this footnote.[10]

[9] The formulas are:

$$g_{\text{total}} = \frac{\sum_{i\in\mathcal{F}_{t+1}} V_{i,t+1}}{\sum_{i\in\mathcal{F}_t} V_{i,t}} - 1; \qquad g_{\text{stayer}} = \frac{\sum_{i\in(\mathcal{F}_t\cap\mathcal{F}_{t+1})} V_{i,t+1}}{\sum_{i\in(\mathcal{F}_t\cap\mathcal{F}_{t+1})} V_{i,t}} - 1,$$
$$g_{\text{entry}} = \frac{\sum_{i\in(\mathcal{F}_{t+1}\setminus\mathcal{F}_t)} V_{i,t+1}}{\sum_{i\in\mathcal{F}_t} V_{i,t}}; \qquad g_{\text{exit}} = -\frac{(1+g_{\text{stayer}})\sum_{i\in(\mathcal{F}_t\setminus\mathcal{F}_{t+1})} V_{i,t+1}}{\sum_{i\in\mathcal{F}_t} V_{i,t}},$$

where $\mathcal{F}_t$ is the universe of firms at time t and V_i is the market capitalization of firm i.

[10] The definitions of g_{stayer}, g_{price}, and g_{shares} are

$$g_{\text{stayer}} = \frac{P_{i,t+1}N_{i,t+1}}{P_{i,t}N_{i,t}} - 1; \qquad g_{\text{price}} = \frac{P_{i,t+1}}{P_{i,t}} - 1; \qquad g_{\text{shares}} = \left(\frac{N_{i,t+1}}{N_{i,t}} - 1\right)(1+g_{\text{price}}),$$

where $P_{i,t}$ is the price per share and $N_{i,t}$ is the number of shares outstanding at firm i period t.

Moreover, we observe the the contribution of the growth in the number of shares issued for cash $g_{\text{shares, cash}}$, and the remainder $g_{\text{shares}} - g_{\text{shares, cash}}$ is due to a combination of stock compensation and stock-financed acquisitions (i.e., "noncash payments"). Using the same logic as for firm entry (see equation 3.9), we define noncash payments as

$$\text{cf adjustment (noncash payments)} \equiv \left(g_{\text{shares}} - g_{\text{shares, cash}}\right) \cdot \text{market capitalization}. \tag{3.11}$$

The breakdown between stock compensation and acquisitions is only available after 2011. Therefore, we assume that acquisition is a constant share $\omega \in (0, 1)$ of noncash payments (stock issuance not associated with cash). We compute ω at the industry year level after 2011, and use the average across years to split stock compensation before 2011.

Adjustment #4: imputation of the tangible share of ambiguous investments.

How should one proceed to allocate the sum of cashflows due to mergers, acquisitions, and IPOs into tangible versus intangible investment? To do so, we use the "ambiguous income" approach introduced in Cooley et al. (1995). In our setup, the idea will be to assume that acquisitions have the same intangible content as other forms of investment. Denoting $\widetilde{\chi} \in (0, 1)$ the tangibility of investment in the rest of investment, we have that

$$\widetilde{\chi} \equiv \frac{\text{cf from investing (capex)}}{\text{cf from investing (capex)} + \text{cf adjustments}\left(\text{investment labor} + \psi \cdot (1-\omega) \cdot \text{noncash payments}\right)},$$

where we discuss the role of ψ shortly. We compute $\widetilde{\chi}$ at the industry year-level. Equipped with this last estimate, we are ready to construct our final variables.

Formulas Having defined the key variables, we now write down the final formulas:

$$\begin{aligned}
\text{tangible investments} = &- \text{cf from investing (capex)}, \\
&- \widetilde{\chi} \cdot \text{cf from investing (acquisition)}, \\
&- \widetilde{\chi} \cdot \text{cf adjustments}\left(\text{net entry} + \omega \cdot \text{noncash payments}\right), \\
\text{intangible investments} = &- \text{cf adjustments}\left(\text{investment labor} + \psi \cdot (1-\omega) \cdot \text{noncash payments}\right), \\
&- (1-\widetilde{\chi}) \cdot \text{cf from investing (acquisition)}, \\
&- (1-\widetilde{\chi}) \cdot \text{cf adjustments}\left(\text{net entry} + \omega \cdot \text{noncash payments}\right), \\
\text{payments to capitalists} = &- \text{cf from financing} \\
&+ \text{cf adjustments}\left(\text{net entry} + \text{noncash payments}\right).
\end{aligned}$$

Combining these formulas, we obtain an expression for capital income

$$\begin{aligned}\text{capital income} =& \text{tangible investments} + \text{intangible investments} + \text{payments to capitalists} \\ =& \text{cf from operations} + \text{cf adjustments (investment labor)} \\ & - (1-\psi)\cdot(1-\omega)\cdot\text{noncash payments},\end{aligned}$$

where the second equality uses the cashflow identity (3.3). The number $\psi(1-\omega)$ corresponds to the share of noncash payments that is unreported in R&D and SG&A. Following changes in accounting standards in 2006, we set $\psi = 0.5$ before 2006 and $\psi = 0$ afterwards.

A few remarks are in order. First, that payments to capitalists are invariant to the calibration $(\widetilde{\chi}, \omega)$. This is because they only represent assumptions on how cashflows are distributed between tangible investments and intangible investments. Second, capital income is invariant not only to the calibration $(\widetilde{\chi}, \omega)$, but also the adjustments for noncash payments and net entry. This is because noncash payments and net entry are purely redistributive flows (from capitalists to tangible investments and to innovators). The only adjustment that affects the level of capital income is the adjustment for expensed investment labor. The idea is that we reclassify line items as capital expenditures, not expenses.

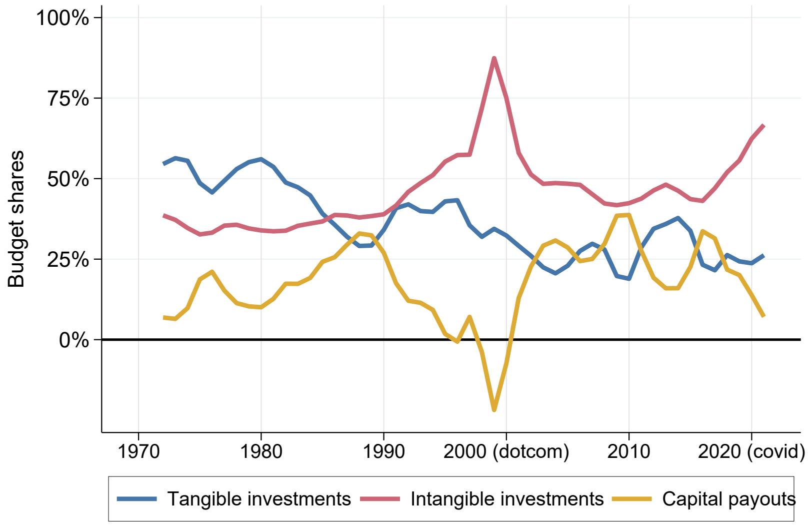
3.3 Results

Figure 2a reports the distribution of capital income over time. Three facts stand out: (1) the tangible investment share declines, (2) the intangible investment share increases, and (3) the payout share is low on average and highly countercyclical.

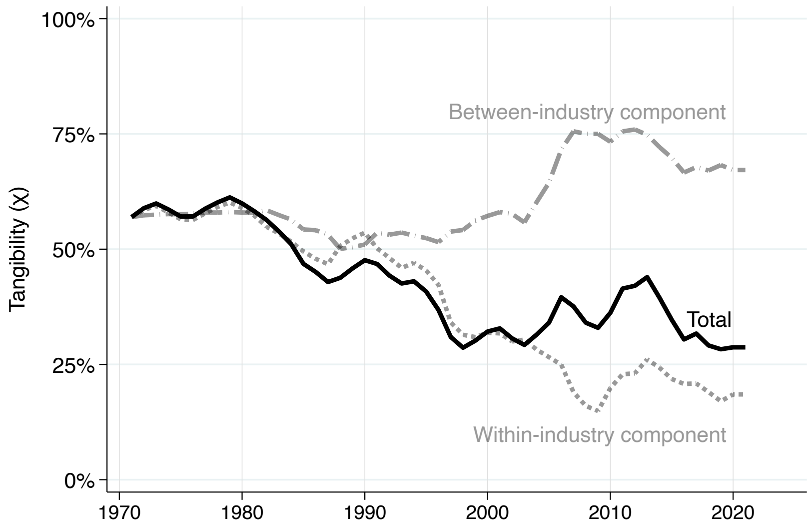
Those three facts are consistent with findings in the literature relating to the stagnation of capital expenditures post-GFC (Gutiérrez and Philippon, 2017; Crouzet and Eberly, 2019), the importance of payments to investment labor (Eisfeldt and Papanikolaou, 2013; Peters and Taylor, 2017; Eisfeldt et al., 2023), and the empirical evidence on the aggregate equity payout yield (Fama and French, 2005; Boudoukh et al., 2007; Fried and Wang, 2019). Zooming in on the tangibility of investment,

Figure 2b shows a large decline in the tangibility of investment, from roughly 60% in the 1970s to 35% post-2000. We further decompose the decline in the tangibility of investment using a shift-share approach. More specifically, we quantify how much the decline in aggregate tangibility is driven by a decline in tangibility within each industry (a *within* component) rather than intangible industries growing bigger relative to intangible industries (a *between* component). We find that most of the effect is driven by the within component, implying that the effect is not driven by changes in the industry composition of public firms.

Table 2 reports the average distribution of capital income over the 1972–2022 period, including a detailed breakdown, and robustness checks. Tangible investments account for 42% of capital over the period, mostly due to direct capital expenditures, but also in part due to acquisition of existing businesses (who themselves are partial tangible). Intangible investments account for 44%. Cash and noncash compensation to investment labor accounts for most of it, but almost 20% comes from payouts associated with cash and noncash payout that arise in acquisitions.



(a) Uses of capital income



(b) Tangibility of investment

Notes: Aggregate tangibility is $\chi \equiv \sum_i w_i \chi_i$ where χ_i denotes tangibility in industry i and w_i denotes the fraction of aggregate investment accounted by that industry. The yearly change in aggregate tangibility can be decomposed into a within and a between components: $\Delta\left(\sum_i w_i \chi_i\right) = \sum_i \left(w_i + \frac{\Delta w_i}{2}\right) \Delta \chi_i + \sum_i \left(\chi_i + \frac{\Delta \chi_i}{2}\right) \Delta w_i$. To obtain the two components plotted in the figure, we start from the aggregate tangibility at time $t = 1971$, and add the cumulative sum of the within and between components, respectively.

Figure 2: Capital income in the corporate sector (1972–2022)

Finally, payments to capitalists account for a mere 14% of capital income over the sample. While net cash payouts to equity holders account for 19% of capital income, they were offset by a 12% equity dilution (i.e., noncash payments). This sample average is skewed by the tech bubble of the 1990s, where payments to capitalists were negative for a few years (i.e., acquisitions and IPOs exceeded dividends and interest payouts, see Fried and Wang, 2019 for a discussion of this fact).

Benchmarking against the National Accounts. In the Appendix, we contrast our results with aggregate data from the National Accounts. Our definition of capital is meant to be (mostly) consistent with the national accounts definition. There are two key differences. First, our treatment of noncash

Table 2: Distribution of capital income (1972–2022 average).

Uses of capital income (%)	Baseline	Robustness			
		$\widetilde{\chi}=0$	$\widetilde{\chi}=1$	$\omega=0$	$\omega=1$
Tangible investments	36	37	45	37	42
Tangible capital expenditures	37	37	37	37	37
Mergers, acquisitions, and IPOs	-1	0	8	0	5
$\chi\cdot$ cash acquisitions	3	0	8	3	3
$\chi(1-\omega)\cdot$ non-cash payments	2	0	5	0	5
$\chi\cdot$ net entry in public universe	-6	0	-6	-3	-3
Intangible investments	46	46	38	42	46
Intangible capital expenditure	38	38	38	39	35
$0.3\cdot$ selling, general, and admin. expenses	24	24	24	24	24
research and development expenses	12	12	12	12	12
$\psi\cdot\omega\cdot$ non-cash payments	2	2	2	4	0
Mergers, acquisitions, and IPOs	8	8	0	3	10
$(1-\chi)\cdot$ cash acquisitions	5	8	0	5	5
$(1-\chi)(1-\omega)\cdot$ non-cash payments	3	5	0	0	7
$(1-\chi)\cdot$ net entry in public universe	1	-6	0	-2	-2
Payout to capitalists	18	18	18	18	18
Net cash equity payout	19	19	19	19	19
− Non-cash payments	-13	-13	-13	-13	-13
− Net entry in public universe	6	6	6	6	6
Net debt payout	6	6	6	6	6

payments is more comprehensive than what the BEA does. It is well understood that the BEA undercounts labor income by missing payments or underestimating their true value (see Zwick, 2022 for a detailed discussion on the topic). In contrast, our approach uses the actual growth in the number of (fully-diluted) shares times the market value at the time to impute the value of payments in stocks. Second, our definition of what an innovator potentially differs from the BEA. The BEA mostly capitalizes expenses on labor in the case of research and development (see Corrado et al., 2009). Instead, we opt for a more comprehensive definition of "investment labor", which includes not only scientists, bu also key managers, entrepreneurs, and early financiers. Appendix Figure A2 plots capital income and enterprise value, both in our sample and in the the Integrated Macroeconomic Accounts. Appendix Figure A3 compares the time variation in the use of capital income in the national accounts versus Compustat.

4 Calibration and counterfactuals

We now calibrate the model and use it to generate counterfactuals. We first look at the effect of a shift towards intangible capitals. We then discuss the effect of a change in corporate taxes.

4.1 Calibrating the model

To bring the model to the data, we first add a few ingredients to make it more realistic : (i) corporate tax (ii) an elastic labor supply between production and investment workers (iii) the productivity of production and investment labor grow at some rate π (balanced growth path).

Firm side. We use the same production side except we now add corporate taxes, that are rebated to the household The firm budget constraint then becomes:

$$D_t = (1 - \tau_{K,t})(Y_t - w_{Y,t}L_{Y,t}) - I_t - w_{H,t}L_{H,t}, \qquad \text{(firm budget constraint')}$$

where $\{\tau_{K,t}\}_{t=1}^{\infty}$ is a sequence of capital income tax rates, or more precisely taxe on payouts.

Household side. To make the model more realistic, we now allow the quantity of production and investment labor to be elastically supplied. More precisely, we assume that the representative household chooses a sequence of consumption, production labor, and investment labor to maximize welfare:

$$U_0 = \max_{\{C_t, L_{Y,t}, L_{H,t}, V_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t \left\{ \frac{1}{1-\gamma} \left(C_t - \frac{L^{1+\sigma}}{1+\sigma} \right)^{1-\gamma} \right\}. \tag{4.1}$$

$$\text{where } L_t \equiv \left((1-\mu)^{-\rho} L_{Y,t}^{1+\rho} + \mu^{-\rho} L_{H,t}^{1+\rho} \right)^{\frac{1}{1+\rho}} \tag{4.2}$$

The parameter $\sigma > 0$ governs the labor supply elasticity. The parameter $\rho > 0$ governs the elasticity of substitution between production and investment labor. The stylized model described in Section 2 corresponds to the special case $\sigma = \rho$ and $\sigma \to \infty$.

The optimality conditions with respect to the supply of labor are:

$$L_t = w_t^{\frac{1}{\sigma}}; \qquad L_{Y,t} = (1-\mu)L_t \left(\frac{w_{Yt}}{w_t} \right)^{\frac{1}{\rho}}; \qquad L_{H,t} = \mu L_t \left(\frac{w_{H,t}}{w_t} \right)^{\frac{1}{\rho}} \qquad \text{(worker foc } L_t)$$

where w_t is defined as L_t. The higher σ is, the more inelastic aggregate labor supply is. The higher ρ is, the lower the elasticity of substitution between production and investment worker. If $\rho \to 0$ (perfect substitution), the wages between production and investment workers are equalized, $w_{H,t} = w_{Y,t}$.

An equilibrium is an initial condition K_0, an allocation $\{L_{Y,t}, L_{H,t}, L_t, I_t, K_t\}_{t \geq 1}$, and prices $\{w_{H,t}, w_{L,t}, w_t, R_t, q_t\}_{t \geq 1}$ that solve the firm problem (firm foc $L_{Y,t}$, firm foc $L_{H,t}$, firm foc I_t, firm foc K_{t+1}) and household problem (worker foc C_t, worker foc L_t).

Calibration. First, we set two parameters externally. We set $\sigma = 2$ to match an aggregate labor supply elasticity of 0.5, consistent with the macro evidence at the business cycle frequency. We then set $\rho = \sigma$, which corresponds to a linearly separable labor supply for production workers and investment workers. The parameter β is chose to match a roughly 7% log return (unlevered), which is the average over our sample. In Appendix C.2, we show that, along a balanced growth path with growth π, the log return is constant at $\log R = -\log \beta + \gamma\pi$, where γ is the household's EIS. Assuming $\gamma = 1$ and

$\pi = 2\%$, we get $\beta = 0.95$. Second, we calibrate the parameter $\chi = 0.41$, which governs the importance of tangible inputs in capital formation using our evidence on the tangibility of investment (2b) .

Third, we calibrate the remaining parameters four parameters (α, θ, δ) internally by targeting two long-run moments (the total investment yield and the labor income share) and one short-run moment (the tax elasticity of investment). The total investment yield is the sum of tangible and intangible investments, as a share of enterprise value. In Appendix A.1, we discuss how this moment has the interpretation of a dilution rate: it measures how much a capitalists that would consume all the capital income would get diluted over time. The second moment is the labor share, which is defined as total payments to labor (to both production and investment labor) as a share of aggregate income.

Table 3: Internally calibrated parameters

Moment	Notation	Formula	Target
Total investment yield	$\frac{I+w_H L_H}{V}$	$(1-\theta)\delta$	13%
Capital share	$\frac{\Pi}{GDP}$	$\frac{\alpha}{1+(1-\chi)\alpha\frac{(1-\theta)\delta}{r+(1-\theta)\delta}}$	33%
Tax elasticity of investment	$\frac{\partial \log(I_0+w_{H,0}L_{H,0})}{\partial \log(1-tau_K)}$	$\frac{r+(1-\theta)\delta}{r+\phi}\frac{1}{\theta}$	4

Notes. α is the capital share in production; θ is the production share in investment; χ is the tangibility of investment; δ is the depreciation rate; r is the return; ϕ is the annual decay of the tax cut.

Note that, together, these two long-run moments can not separately identify $1-\theta$ from δ. The reason is that a high investment yield can be due to the fact that capitalists extract little rents from investment (θ is low) or that the quantity of investment is high (δ is high). Income shares alone can not distinguish between these two cases. Guided by the earlier intution that θ effectively governs the elasticity of capital supply in the short-run (as in the q-theory of investment), we use a short-run moment for our last empirical target.

We draw on evidence from Chodorow-Reich, Smith, Zidar and Zwick (2024) on the empirical response of US firms to the 2017 Tax Cuts and Jobs Act. We focus on their result that uses tax files for a sample of roughly $N = 7000$ (nonfinancial, non-passthrough domestic firms). The variable of interest is log investment change (post-TCJA versus pre-TCJA). The authors estimate that a 1% decline in the corporate income tax rate leads to a roughly 4% decline in investment (see their Table 3, columns 2 and 3). We interpret their estimates as a partial equilibrium short-run response in the model (i.e., holding prices w_Y, w_H, R constant).

In Appendix 4, we map their regression coefficient to a simple formula that is the product of two terms. We show that a tax cut (with an annual decay rate of τ) implies a short run response of log total investment of $\frac{r+(1-\theta)\delta}{r+\phi}\frac{1}{\theta}$ (see Table 3). The first terms accounts for the fact that (i) capital payout is levered to the tax rate and (ii) the tax cut is not fully permanent. The second term is $\frac{1}{\theta}$, which governs the short-run elasticity of capital supply in partial equilibrium.

Using these short- and long-run targets, we obtain the following model calibration:

$$\alpha = 0.38, \qquad \theta = 0.3, \qquad \delta = 0.19, \qquad \chi = 0.41. \qquad \text{(preferred calibration)}$$

4.2 Model experiment: Shift towards intangibles

We now conduct an experiment where we quantify the long-run effect of a decline in the tangibility of investment χ from 0.54 to 0.29 (i.e., a 25 pp. decline around the baseline calibration), in line with what we have seen in the data over the 1972-2022 period (see Figure 2b).

Table 4: Distributional effect (shift towards intangibles).

Variable	Symbol	Baseline (pp.)	1972→2022
Panel A – Expenditures			
Consumption	$Y - I$	76	-0
Tangible investments	I	10	-6
Intangible investments	$w_H L_H$	14	6
Panel B – Income			
Capital income	$Y - w_Y L_Y$	33	0
Labor income (production)	$w_Y L_Y$	53	-6
Labor income (investment)	$w_H L_H$	14	6

First, Table 4 reports the effect of rising intangibility on the composition of GDP (see equation 3.1 for national accounting definitions). On the expenditure side (Panel A), there is a rise in intangible investments (6 pp. of GDP) offset by a decline in tangible investments (-6 pp. of GDP). On the income side (Panel B), we see decline in the production labor income share 53 pp. compensated by a rise in the investment labor income share (53 pp.).

Effect on wage gap. A decline in χ corresponds to a rise in labor demand for one type of labor (i.e., investment labor). How does that translate into changes in relative wages? In the counterfactual, we obtain a large response of the wage premium by $\Delta(w_H/w_Y)/(w_H/w_Y) = +33\%$. To understand the sources of the rise in the wage premium, we use the following decomposition:

$$\mathrm{d}\log\frac{w_H}{w_Y} = \frac{\rho}{1+\rho}\,\mathrm{d}\log\frac{w_H L_Y}{w_Y L_Y} = -\frac{\rho}{1+\rho}\frac{\mathrm{d}\chi}{1-\chi}$$

The first equality uses labor supply (worker foc L_t) to express the change in relative wage in terms of the change in wage bills, which depends on the occupation-level labor supply elasticity ρ. If it was perfectly elastic ($\rho = 0$), we would see no change in relative wages. In the baseline calibration, labor is moderately elastic, with $\frac{\rho}{1+\rho} = 0.67$. The second equality uses the long-run relative marginal products of labor (2.2) $\frac{-\mathrm{d}\chi}{1-\chi} = 54\%$.

In our simulation, the wage premium of investment labor rises in the long-run, due to the fact that they effectively become more scarce. What this means is that we should observe rising wage polarization in the labor market, where demand for workers with skills that are important for capital formation should lead to higher relative wages (as discussed in Section 2.3). This pattern aligns with findings from the literature on U.S. wage inequality since the 1980s, which document that occupations requiring "abstract" skills have seen a relative increase in wages compared to those relying on "routine" skills (see, e.g., Autor and Dorn, 2013).

Quantitatively, our model predicts a 36 log point increase in wage gap and a 18 log point increase in employment following a decline in χ by 54%. How does it compare to the rise in the wage gap in the data? To answer this question, Figure 3 plots the employment and wage indices for production labor and investment workers using data from the Current Population Survey. We define "investment labor" as occupations in managerial and professional specialty occupations while "production labor" as occupations in technical, sales, and administrative support occupations, service occupations, and production occupations. We find an increase in the wage premium of 17 log points, which is roughly half of what the model predicts.

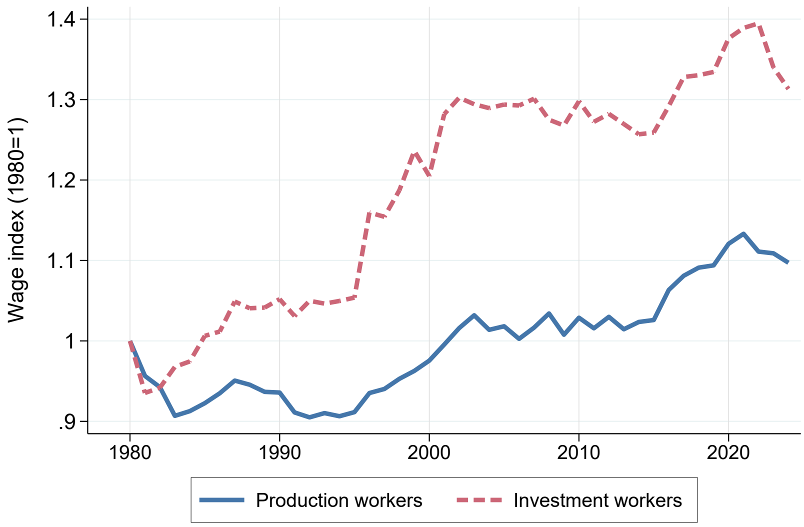


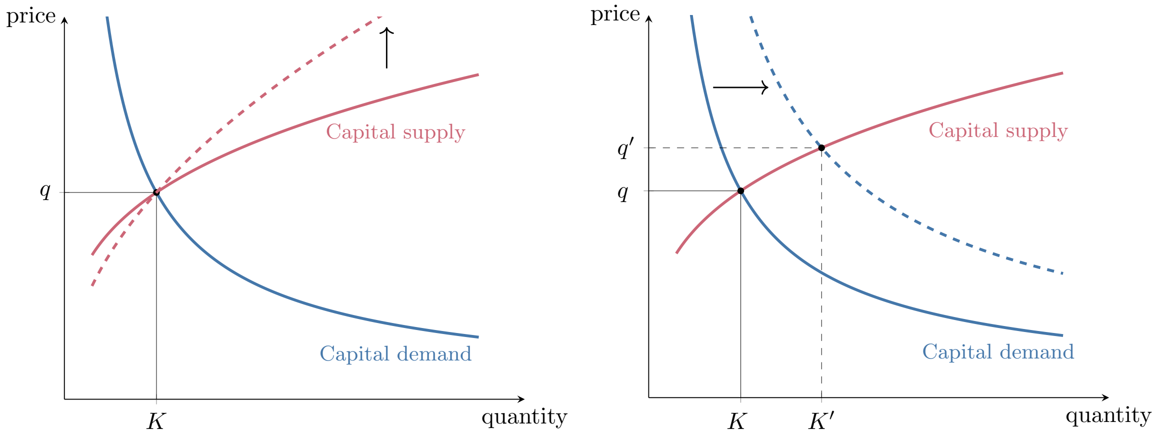
Figure 3: Wage a index for production and investment workers

Notes: The figure plots the wage index within production and investment workers. This index is obtained by chaining the wage growth within production and investment workers every year. The wage growth for production workers (resp. investment workers) in a given year is obtained by aggregating the wage growth across sub-occupations using a Fisher Price Index.

Figure 4b shows the long-run effect of rising intangibles on the capital supply curve, which amounts to a counter-clockwise rotation of the supply curve.[11] Notice that capital supply becomes much less elastic, due to the growing importance of imperfectly-elastic labor in capital formation.

Quantitatively, varying the value of χ in the model, while holding all the other parameters constant, has a large effect on both the short- and long-run capital supply elasticities. Table 5 reports the model-implied elasticities. The baseline model has low elasticities compared to the case of full tangibility (i.e.,

[11]Note that a change in χ affects the long-run equilibrium Y due to an endowment effect. For visual purposes, we construct Figure 4b by showing a rotation around the initial equilibrium, which amounts to consider a joint change in χ and z_H that ensure no change in the long-run (q, K).



(a) Model experiment, rise of intangible investment

(b) Model experiment, permanent tax cut

Figure 4: Counterfactual exercises

setting $\chi = 1$ as in the standard q-theory of investment). In the short-run it is 0.14 versus 0.43, and in the long-run it is 1.54 versus $+\infty$. Simulating the transition from 1972 to 2022 (i.e., rise in intangibles), we obtain a decline in both the long-run elasticity by more than half.

Table 5: Capital supply elasticity.

	Formula	Baseline	Q-theory $\chi = 1$	1972 $\chi = 0.54$	2022 $\chi = 0.29$
Short-run	$\frac{\partial \log K_{t+1}}{\partial \log q_t}$	0.14	0.43	0.17	0.11
Long-run	$\frac{\partial \log K}{\partial \log q}$	1.54	$+\infty$	2.23	1.1

While we have focused on the effect of an aggregate change in χ on the elasticity of capital, our model also predicts that, in the cross-section, industries with a higher share of intangibility tend to have a lower capital supply elasticity. As a model validation exercise, we examine the relationship between short-run capital elasticity and tangibility at the industry-level. Assuming that labor markets are perfectly segmented, we can map our theory directly to the industry evidence. To estimate the short-run investment-q elasticity $\frac{\partial \log(I_t + w_{H,t} L_{H,t})}{\partial \log q_t}$, we leverage cross-industry variation in total investment and valuations. We focus on $J = 33$ broad industry groups over 1972-2022. As an empirical proxy for $\log q_{j,t}$ we use log enterprise value $\log V_{j,t}$. The idea is that annual industry-level fluctuations in market values are mostly driven by changes in the valuation of capital q_t, rather than changes in capital K_t. We estimate an industry-specific "investment-q elasticity" φ_j

$$\Delta \log(I_{j,t} + w_{j,t} L_{H,j,t}) = \text{controls}_{j,t} + \varphi_j \cdot \Delta \log V_{j,t} + v_{j,t} \tag{4.3}$$

Figure 5 plots the average tangibility of investment over the sample for each industry, as well as its investment-q elasticity. Despite the limited sample, we find a significant positive relationship, which is consistent with the prediction from the model.

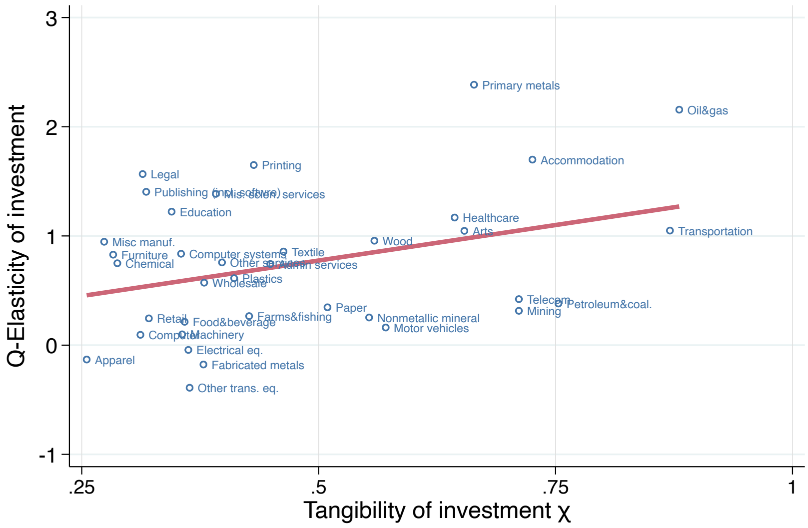


Figure 5: Investment-q elasticity and tangibility of investment.

Notes: On the x-axis, the industry-level tangibility of investment is the (unweighted) 1972-2022 average χ_j. On the y-axis, we report estimated coefficient φ_j, which captures the elasticity of total investment to firm value (a proxy for q). The red line reports the predicted relationship using all data points (unweighted).

4.3 Model experiment: Capital taxation

We now consider the effect of a permanent tax cut $\mathrm{d}\log(1-\tau_K) > 0$. This can be seen as a shift in the demand for capital, as represented in Figure 4b. As usual in supply-demand systems, a positive demand shock will increase prices and quantities. When the supply curve is horizontal (as in the ngm), then the demand shock is entirely absorbed by quantities K. But in the baseline, long-run capital supply is imperectly elastic due to finite labor supply. As a result, a demand shock, in this case a tax cut, leads to a rise of both prices q_t and quantities K_t.

To set ideas, consider a permanent decline in τ_K. Using a standard comparative statics approach and the long-run demand and supply elasticities in the model, we obtain

$$\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau_K)} = \frac{1}{\text{demand elasticity} + \text{supply elasticity}} = 0.9,$$
$$\frac{\mathrm{d}\log q}{\mathrm{d}\log(1-\tau_K)} = \frac{\text{supply elasticity}}{\text{demand elasticity} + \text{supply elasticity}} = 0.6.$$

The shock is absorbed through both a higher valuation of capital q *and* a rise in capital formation K. As a benchmark, in the NGM, the response of capital would be roughly twice as high (1.9) with no response of valuations.

More generally, a shift towards intangibles means that the market increasingly clears via higher valuations rather than actual capital formation. As we discuss next, this imperfect pass-through of demand shocks to quantities has important effects on the incidence of capital taxes (i.e., who wins and loses in terms of welfare). Intuitively, a jump q induced by a tax cut will benefit investment labor (via higher wages) and initial capitalists (via a revaluation gain).

Incidence of capital taxes. We first consider an arbitrary perturbation $\{d\tau_{K,t}\}_{t=0}^{\infty}$ around the undistorted steady-state. Which factors of production win and/or lose in response to this shock? We follow Fagereng et al. (2024) and apply the envelope condition on the household value function (4.1), combined with the Euler condition worker foc C_t, to obtain the total welfare effect of the change in prices induced by the tax shock.

Proposition 4.1. *The equilibrium welfare effect, in units of $t = 0$ consumption, associated with the perturbation $\{d\tau_{K,t}\}_{t=1}^{\infty}$ around the undistorted steady-state, is given by*

$$\textit{Welfare Gain} = \underbrace{\sum_{t=1}^{\infty} R^{-t}(dw_{Y,t})L_Y}_{\textit{welfare gain (production labor)}} + \underbrace{\sum_{t=1}^{\infty} R^{-t}(dw_{H,t})L_H}_{\textit{welfare gain (investment labor)}} \underbrace{+ dV_0 + \sum_{t=1}^{\infty} R^{-t}(dR_t)V}_{\textit{welfare gain (capitalists)}}$$

Proposition (4.1) decomposes the welfare effect into the contribution of changes in prices for the three factors of production: production labor, investment labor, and capital. For labor, the welfare effect captures the fact that the tax shock affects the path of wages $\{dw_{Y,t}, dw_{H,t}\}_{t=1}^{\infty}$. For capitalists, there is an initial revaluation of wealth dV_0 as well as the contribution of changes in forward returns $\{dR_t\}_{t=1}^{\infty}$. The total (private sector) welfare effect is equal to the present value of the change in the tax rate times capital income (i.e., the mechanical effect of taxes on profits holding everything else constant): Welfare Gain $= \sum_{t=1}^{\infty} R_{0\to t}^{-1}(d\tau_{K,t})\Pi_t$.

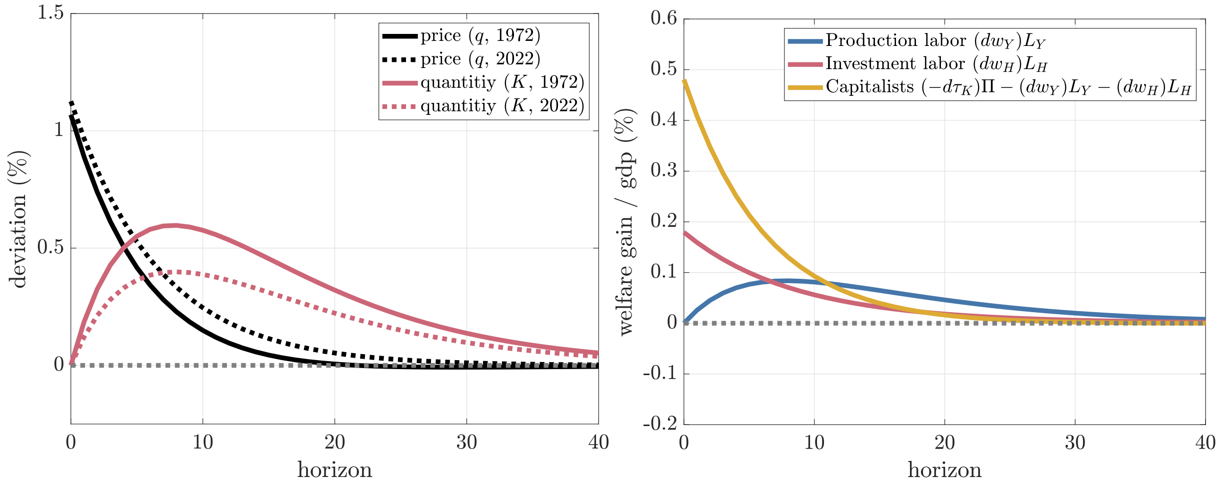
Experiment. We now simulate a 1% cut around the steady-state, which decays at an annual rate of $1 - \phi = 0.9$. We consider the case of a small-open economy, where forward returns are constant (i.e., $dR_t = 0$). Throughout this section, our baseline environment is a small open economy, where we simulate deviations induced by taxes while imoposing $dR_t = 0$ (the interest rate is not affected by the shock, but wages are).[12] This simplifies the formula for the welfare effect for capitalists in Proposition (4.1), which becomes only the revaluation gain dV_0 (i.e., the rise in their wealth due to the fact that future post-tax capital income has increased).

Figure 6a plots the response of capital accumulation and valuations over a 40 year period after the shock. We show the dynamics implied by the model using the 1972 and 2022 calibrations to describe how the shift towards capital affects the response of the economy. In both cases, the value of capital jumps on impact, but it takes several years for the stock of capital to peak. Notice, however, that in the intangible economy (2022 calibration), the stimulative effect of the tax cut is much lower, consistent with the earlier calculations regarding the supply elasticity of capital (see Table 5).

How does the general equilibrium (GE) response of investment respond to the partial equilibrium (PE) target used for calibration? In PE, we target a high elasticity, but in a GE experiment where the tax applies to all of capital income (which represents 4% of GDP in the baseline), we expect a much weaker response due to rising wages and interest rates.

Table 6 reports the elasticity of total investment to the tax shock in PE (our calibration target), as well as the corresponding elasticity in a small open economy (the baseline), and in GE. We find that

[12] In terms of the implied dynamics dq_t, dK_t, assuming a "small open economy" is this is equivalent to setting the EIS to $\gamma = 0$, which implies $R_t = \beta^{-1}$.



(a) Capital and valutations

(b) Welfare effects

Figure 6: Model experiment: Tax cut

Table 6: Short-run response to a tax shock.

Tax elasticity of:	Investment $(I + w_H L_H)$	Investment wages (w_H)	Valuation (q)
Partial equilibrium	4	0	1.1
Small open economy (baseline)	0.96	0.64	0.55
General equilibrium	0.36	0.24	0.21

Notes. The short-run tax elasticity of a variable X_t is the response $\mathrm{d}\log X_1$ in response to a tax shock $\mathrm{d}\tau_{k,t} = (1-\phi)^t\, \mathrm{d}\tau_K$ ($\phi = 0.1$ annually). "Partial equilibrium" is the equilibrium deviation with the restriction $\mathrm{d}w_Y = \mathrm{d}w_H = \mathrm{d}R = 0$; "small open economy" is the equilibrium deviation with the restriction $\mathrm{d}w_Y = \mathrm{d}w_H = 0$; "general equilibrium" is the equilibrium deviation where all prices adjust (EIS is set to $\gamma = 1$)).

rising wages reduce the tax elasticity of investment nearly fourfold. The second column reports the tax elasticity of investment wages: compared to the PE environment (where wages are fixed), in a small open economy the elasticity is large at 0.64. Finally, the GE case, where interest rates $\mathrm{d}R_t$ also respond, has an even lower tax elasticity of investment. The response of interest rates is governed by the household EIS $\frac{1}{\gamma}$. Intuitively, in the case where $\gamma \to \infty$ (Leontief preferences), aggregate capital does not adjust because household are not willing to substitute across time periods. We report the case $\gamma = 1$ (log utility) and the resulting tax elasticity of investment nearly ten times lower than in PE.

Welfare effects. The welfare effect for workers works through changes in their wage, which are themselves fully pinned down by the path of the state and co-state variables $\{q_t, K_t\}_{t=0}^{\infty}$. A rise in capital K benefits both types of workers, via their complementarity with existing capital, but the value of capital q directly affects the marginal product of investment labor. Figure 6b plots the resulting path of (undiscounted) welfare effects for both types of labor, expressed as a share of steady-state GDP. While the tax shock appears to benefit both types of labor equally, it is worth pointing out that production labor accounts for roughly 4 times more labor income than investment labor in steady-state (see Table 4).

The welfare effect for capitalists can be decomposed as the present value of higher future payouts:

$$\mathrm{d}V_0 = \sum_{t=1}^{\infty} R^{-t}\Big((-\,\mathrm{d}\tau_{K,t})\Pi - (\mathrm{d}w_{Y,t})L_Y - (\mathrm{d}w_{H,t})L_H\Big). \qquad \text{(Revaluation gain)}$$

Notice that those higher payouts are very front-loaded, especially compared to the back-loaded increase in production worker wages (see Figure 6b). This is because wages rise slowly with capital accumulation, while the benefit for capitalists on post-tax capital income is immediate.

Table 7 reports the share of private welfare gains that accrue to each factor of production. In the baseline, 20% of the tax incidence falls on production workers, while 26% falls on investment labor and 54% on capitalists. Note that this stands in sharp contrast with the NGM, where 100% of the incidence falls on productions workers. This is because, when capital is fully elastic ($\theta = 0$), there is no revaluation effect since $q = 1$ at all times.

Table 7: Incidence of capital taxes (share of private sector welfare gains).

Factor	Baseline	NGM	Q-Theory	1972	2022
		$\theta = 0, \chi = 1$	$\chi = 1$	$\chi = 0.54$	$\chi = 0.29$
Production labor	20	100	50	26	17
Investment labor	26	0	0	22	30
Capitalists	54	0	50	52	53

To understand the difference between the baseline and the NGM, it is useful to consider an intermediary model (i.e., the q-theory special case), where we impose ful tangiblity $\chi = 1$ while keeping a positive capital share $\theta = 0.3$ as in the baseline calibration. In that case, we a slightly lower capitalist share. We also run the same model experiment in the low- and high-intangible calibrations that correspond roughly to 1972 and 2022. Notice that the shift towards intangibles has led to a reallocation of the incidence of capital taxes, away from production labor and towards investment labor and capitalists.

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A Appendix for Section 2

A.1 Capital rents

We now describe the rents associated with capital as well its decay over time

Definition. The firm extracts rents in the model, in the sense of Ricardo (i.e., workers are paid their marginal product but below their average product). We define production and investment rents associated to the ownership of capital as:

$$\text{production rents} \equiv L_{Y,t} \cdot \left(\frac{Y_t}{L_{Y,t}} - \frac{\partial Y_t}{\partial L_{Y,t}} \right) = \alpha Y_t$$

$$\text{investment rents} \equiv \left(I_t^{\chi} L_{H,t}^{1-\chi} \right) \cdot \left(\frac{H_t}{I_t^{\chi} L_{H,t}^{1-\chi}} - \frac{\partial H_t}{\partial (I_t^{\chi} L_{H,t}^{1-\chi})} \right) = \theta q_t H_t.$$

Production rents are the difference between the average and marginal product of production labor. Production rents represent a constant share α of gross output, as in the NGM. We define investment rents similarly, but where the input to investment is the bundle of tangible and intangible investments. Capital formation also creates rents that accrue to existing owners of capital, again because the bundle of tangible and intangible investment is paid below its average product. Using firm optimization we have that a share θ of the value of capital formation accrues to existing capitalists.

Dilution rate. The financial return of owning the representative firm is the payout yield plus the growth of firm value:

$$R_{t+1} \equiv \frac{D_{t+1}}{V_t} + \frac{V_{t+1}}{V_t}.$$

Because of constant return to scales in the production and investment function. The return of owning the representative firm is the same as the return of owning one unit of productive capital.

Using (firm budget constraint), we can alternatively write the return of owning the representative firm as:

$$R_{t+1} = \frac{\Pi_{t+1}}{V_t} + \left(1 - \frac{I_{t+1} + w_{H,t+1} L_{H,t+1}}{V_{t+1}} \right) \frac{V_{t+1}}{V_t}, \tag{A.1}$$

where the first term is now the capital income yield: the owner's share of current profits. The second term accounts for the "diluted" share of firm growth. The idea is that, in order to pay out all of capital income, a firm would need to give away a share $\frac{I + w_H L_H}{V}$ of its ownership to outside investors in order to finance its growth. Alternatively, this dilution rate can be seen as the extent to which a capital owner consuming its share of capital income would see its ownership share of the corporate sector declines over time.[13]

Using the definition of returns and solving forward, we can obtain the value of the firm as the

[13] Our concept of dilution rate is related to Gârleanu et al. (2016), who stresses that even an owner of the corporate sector consuming its share of dividend income would see its ownership share of the public corporate sector declines due to IPOs.

present value of future dividends, or, alternatively, as the present value of diluted capital income:

$$V_0 = \sum_{t\geq 1} R_{0\to t}^{-1} D_t = \sum_{t\geq 1} R_{0\to t}^{-1} \left(\prod_{1\leq s<t} \left(1 - \frac{I_s + w_{H,s} L_{H,s}}{V_s} \right) \right) \Pi_t, \tag{A.2}$$

This second equality makes it clear that existing capitalists (those that own capital at $t = 0$) do not own all of future capital income. They own a share that decays over time, because they must continuously re-invest to maintain the stock of capital. In the steady-state of our model with no growth, this dilution rate is exactly $\delta(1-\theta)$. In a balanced growth path with growth rate π (see Appendix C.2), this dilution rate is $\delta - \theta(\delta + \pi)$. Note that the dilution rate decreases with θ: the fact that capital owners earn some rents associated with capital formation ($\theta > 0$) makes capital formation less dilutive for capital holders relative to the neoclassical model.

A.2 Proofs

Proof of Lemma 2.1. This simply obtains by solving for K in (capital supply) and (capital demand), respectively □

Lemma A.1. *Log-linearizing the model gives the following backward-looking and forward-looking equation for the quantity and price of capital as a function of productivity and impatience shocks:*

$$\begin{aligned}
\mathrm{d}\log K_{t+1} &= \left(1 - \delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi}\right) \mathrm{d}\log K_t + \delta\frac{\theta}{1-(1-\theta)\chi} \mathrm{d}\log z_H + \delta\frac{(1-\theta)\chi}{1-(1-\theta)\chi} \mathrm{d}\log q_t \\
\mathrm{d}\log q_t &= -\mathrm{d}\log \beta_t + \frac{r + (1-\theta)\delta}{1+r} \mathrm{d}\log z_{Y,t+1} + \frac{\theta\delta}{1+r}\frac{1}{1-(1-\theta)\chi} \mathrm{d}\log z_{H,t+1} \\
&\quad - \left(\frac{r+(1-\theta)\delta}{1+r}(1-\alpha) + \frac{\theta\delta}{1+r}\left(1 - \frac{\theta}{1-(1-\theta)\chi}\right)\right) \mathrm{d}\log K_{t+1} \\
&\quad + \frac{1 - \delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi}}{1+r} \mathrm{d}\log q_{t+1}
\end{aligned}$$

Proof of Lemma A.1. Log-linearizing (capital formation) and (firm foc I_t) gives

$$\begin{aligned}
\mathrm{d}\log K_{t+1} &= (1-\delta)\,\mathrm{d}\log K_t + \delta\,\mathrm{d}\log H_t \\
\mathrm{d}\log H_t &= \mathrm{d}\log z_{H,t} + \theta\,\mathrm{d}\log K_t + (1-\theta)\chi\,\mathrm{d}\log I_t \\
\mathrm{d}\log I_t &= \mathrm{d}\log q_t + \mathrm{d}\log H_t
\end{aligned}$$

Combining the last two equations gives

$$\mathrm{d}\log H_t = \frac{1}{1-(1-\theta)\chi}\left(\mathrm{d}\log z_{H,t} + \theta\,\mathrm{d}\log K_t + (1-\theta)\chi\,\mathrm{d}\log q_t\right) \tag{A.3}$$

Plugging into the first equation gives the following log-linearized backward-looking equation for K_t

$$\begin{aligned}
\mathrm{d}\log K_{t+1} &= (1-\delta)\,\mathrm{d}\log K_t + \frac{\delta}{1-(1-\theta)\chi}\left(\mathrm{d}\log z_{H,t} + \theta\,\mathrm{d}\log K_t + (1-\theta)\chi\,\mathrm{d}\log q_t\right) \\
&= \left(1-\delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi}\right)\mathrm{d}\log K_t + \delta\frac{\theta}{1-(1-\theta)\chi}\,\mathrm{d}\log z_H + \delta\frac{(1-\theta)\chi}{1-(1-\theta)\chi}\,\mathrm{d}\log q_t
\end{aligned}$$

We now find the log-linearized forward equation for q_t. Start from (firm foc K_{t+1})

$$R_t K_{t+1} q_t = \alpha Y_{t+1} + ((1-\delta)K_{t+1} + \theta H_{t+1})q_{t+1}$$

which gives

$$\begin{aligned}
\mathrm{d}\log R + \mathrm{d}\log q_t + \mathrm{d}\log K_{t+1} = {} & \frac{r+(1-\theta)\delta}{1+r}\,\mathrm{d}\log Y_{t+1} + \frac{1-(1-\theta)\delta}{1+r}\,\mathrm{d}\log q_{t+1} \\
& + \frac{1-\delta}{1+r}\,\mathrm{d}\log K_{t+1} + \frac{\theta\delta}{1+r}\,\mathrm{d}\log H_{t+1}
\end{aligned}$$

Substituting out $\mathrm{d}\log Y_{t+1}$ and $\mathrm{d}\log H_{t+1}$ using (output production) and (A.3), respectively, yields

$$\begin{aligned}
\mathrm{d}\log R + \mathrm{d}\log q_t + \mathrm{d}\log K_{t+1} = {} & \frac{r+(1-\theta)\delta}{1+r}\left(\mathrm{d}\log z_{Y,t+1} + \alpha\,\mathrm{d}\log K_{t+1}\right) + \frac{1-(1-\theta)\delta}{1+r}\,\mathrm{d}\log q_{t+1} \\
& + \frac{1-\delta}{1+r}\,\mathrm{d}\log K_{t+1} \\
& + \frac{\theta\delta}{1+r}\frac{1}{1-(1-\theta)\chi}\left(\mathrm{d}\log z_{H,t+1} + \theta\,\mathrm{d}\log K_{t+1} + (1-\theta)\chi\,\mathrm{d}\log q_{t+1}\right)
\end{aligned}$$

Log-linearizing (worker foc C_t) with a potentially time varying impatience parameter β_t, we get

$$\mathrm{d}\log R_{t+1} = -\,\mathrm{d}\log \beta_{t+1} + \gamma\,\mathrm{d}\log C_{t+1}/C_t$$

We consider the case $\gamma \to 0$ to simplify (or open economy). Re-arranging give the forward-looking equation for q_t:

$$\begin{aligned}
\mathrm{d}\log q_t = {} & \mathrm{d}\log \beta_{t+1} + \frac{r+(1-\theta)\delta}{1+r}\,\mathrm{d}\log z_{Y,t+1} + \frac{\theta\delta}{1+r}\frac{1}{1-(1-\theta)\chi}\,\mathrm{d}\log z_{H,t+1} \\
& - \left(\frac{r+(1-\theta)\delta}{1+r}(1-\alpha) + \frac{\theta\delta}{1+r}\left(1-\frac{\theta}{1-(1-\theta)\chi}\right)\right)\mathrm{d}\log K_{t+1} \\
& + \frac{1-\delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi}}{1+r}\,\mathrm{d}\log q_{t+1}
\end{aligned}$$

which concludes the proof. □

Note that log-linearized equilibrium can be written as a system of ODEs:

$$\begin{pmatrix} \mathrm{d}\log K_{t+1} \\ \mathrm{d}\log q_t \end{pmatrix} = (I+A)\begin{pmatrix} \mathrm{d}\log K_t \\ \mathrm{d}\log q_{t-1} \end{pmatrix} + B\begin{pmatrix} \mathrm{d}\log \beta_t \\ \mathrm{d}\log z_{Y,t} \\ \mathrm{d}\log z_{H,t} \end{pmatrix}$$

$$A \equiv \begin{pmatrix} -\delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi} & \delta\frac{(1-\theta)\chi}{1-(1-\theta)\chi} \\ \frac{(r+(1-\theta)\delta)(1-\alpha)+\theta\delta\left(1-\frac{\theta}{1-(1-\theta)\chi}\right)}{1-\delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi}} & \frac{1+r}{1-\delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi}}-1 \end{pmatrix}$$

$$B \equiv \begin{pmatrix} 0 & 0 & \delta\frac{\theta}{1-(1-\theta)\chi} \\ -\frac{1+r}{1-\delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi}} & -\frac{r+(1-\theta)\delta}{1-\delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi}} & -\frac{\frac{\delta\theta}{1-(1-\theta)\chi}}{1-\delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi}} \end{pmatrix}$$

The eigenvalues of the matrix A discipline the speed at which the system converge to its new steady-state after a permanent change in the time preference parameter β, or TFP shocks z_Y and z_H.

A.3 Model extension with multiple capital stocks

Suppose that there are $n = 1, \ldots, N$ capital stocks, each of which has its own accumulation technology $(\theta_n, \chi_n, \delta_n)$. The different capital stocks are aggregated according to a CRS Cobb-Douglas aggregator:

$$H_{n,t} = z_{H,n,t} K_{n,t}^{\theta_n}\left(I_{n,t}^{\chi_n} L_{H,n,t}^{1-\chi_n}\right)^{1-\theta_n}, \qquad \text{(capital formation - type } n\text{)}$$

$$K_{n,t+1} = (1-\delta_n)K_{n,t} + H_{n,t}, \qquad \text{(capital accumulation - type } n\text{)}$$

$$K_t = \prod_{n=1}^{N} K_{n,t}^{\varphi_n}. \qquad \text{(capital aggregator)}$$

All the other model equations remain unchanged, which means that the baseline model is nested with $N = 1$.

A.4 Equivalence to multi-sector model

We now show that the production technology (see Section 2.1) can be re-written as a multi-sector economy that produces that has three factors of production (K, L_Y, L_H) and three sectors that produce, respectively, a final good consumption C and a capital good H. We start with three equations from the model:

$$C = Z_Y K^\alpha L_Y^{1-\alpha},$$

$$I = Z_Y K^\alpha L_Y^{1-\alpha},$$

$$H = Z_H K^\theta I^{\chi(1-\theta)} L_H^{(1-\chi)(1-\theta)}.$$

We can instead express the system as

$$C = Z_Y K^\alpha L_Y^{1-\alpha},$$
$$H = Z_H Z_Y^{\chi(1-\theta)} K^{\theta+\alpha\chi(1-\theta)} L_Y^{(1-\alpha)\chi(1-\theta)} L_H^{(1-\chi)(1-\theta)}.$$

Notice that the capital good production function is constant return to scale in (K, L_Y, L_H). A decline in tangibility χ implies a greater importance of investment labor L_H at the expense of production labor and capital (K, L_Y).

A.5 Employment gap between production and investment workers

Figure A1 plots an employment index for production and investment workers. Similarly to the growth in wage index, the growth in employment index is constructed using a Fisher price index. Note that, by definition, the growth in employment index times the growth in wage index corresponds to the growth in payrolls.

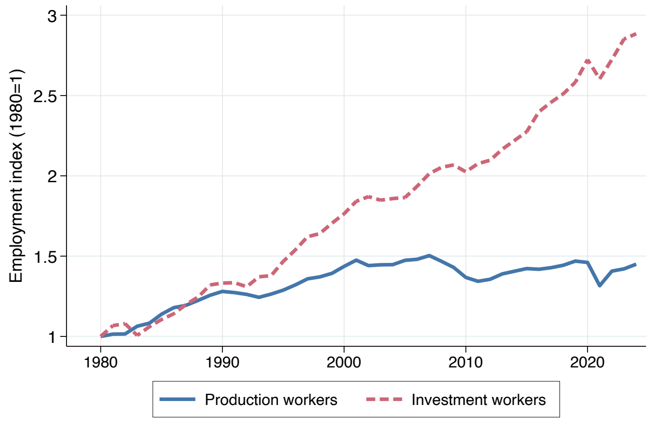


Figure A1: Employment index for production and investment workers

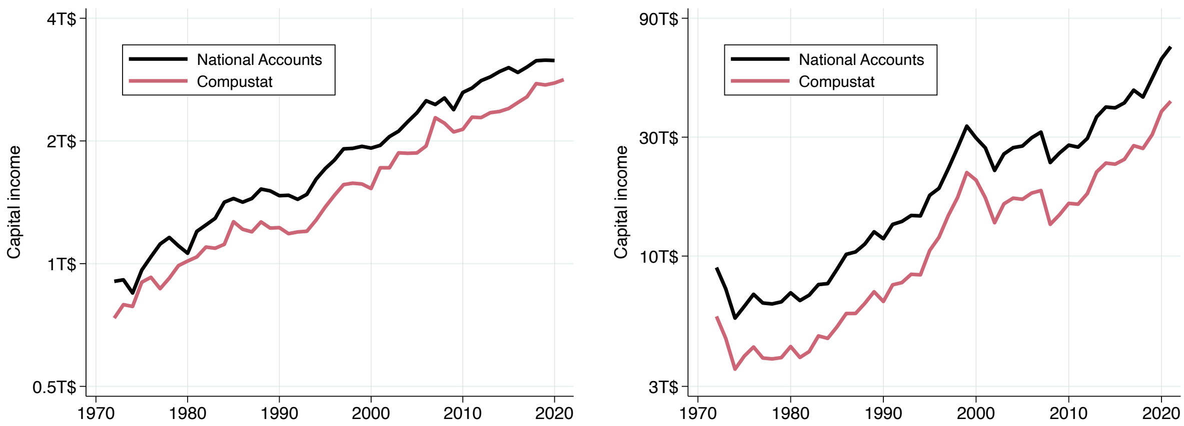
Notes: The figure plots the wage index within production and investment workers. This index is obtained by chaining the wage growth within production and investment workers every year. The wage growth for production workers (resp. investment workers) in a given year is obtained by aggregating the wage growth across sub-occupations using a Fisher Price Index.

B Appendix for Section 3

B.1 Comparison with National Accounts

Appendix Figure A2 plots capital income and enterprise value, both in our sample and in the the Integrated Macroeconomic Accounts (Table S.5.a). Note that our sample accounts for roughly two-thirds of nonfinancial corporate sector gross capital income over the period, with a slight upward trend.

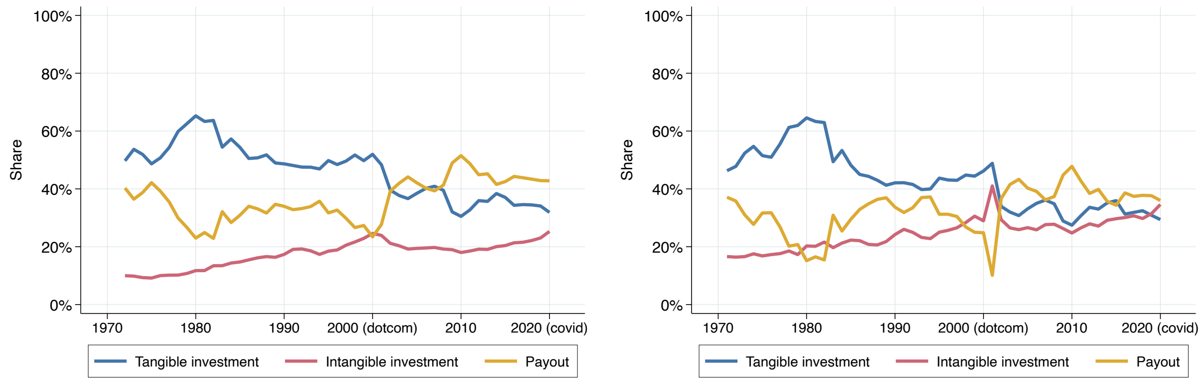
Figure A3 reports the use of capital income for entire U.S. economy using data from the national accounts (BEA). While Panel A3a reports the data aggregated across all industries, Panel A3b reweights the BEA data within industry cells to mimic the distribution of capital in Compustat, showing that part of the difference with the distribution of capital income in Compustat (as reported in Figure 2a)



(a) Capital income (b) Enterprise value

Figure A2: Sample validation against National accounts data (1970–2022).

comes from the difference in industry composition across the two samples. Still, the figure due to the difference between public firms and private firms, as well as differences in the definition of intangible investment in our paper relative to the BEA classification.



(a) Economy-wide (b) Reweighting industries to match public firms

Figure A3: Distribution of capital income (BEA)

C Appendix for Section 4

C.1 Proofs

Proof of Proposition 4.1. The Lagrangian associated with the household problem (4.1) is

$$\mathcal{L}_0 = \sum_{t=1}^{\infty} \beta^t U(C_t, L_t) + \sum_{t=1}^{\infty} \beta^t \lambda_t \Big(C_t + V_t - w_{Y,t} L_{Y,t} - w_{H,t} L_{H,t} - R_t V_{t-1} \Big)$$

$$\text{where } U(C, L) \equiv \frac{1}{1-\gamma} \left(C_t - \frac{L_t^{1+\sigma}}{1+\sigma} \right)^{1-\gamma}.$$

Applying the envelope theorem, we have

$$\begin{aligned}
\mathrm{d}U_0 = \mathrm{d}\mathcal{L}_0 &= \sum_{t=1}^{\infty} \beta^t \lambda_t \Big((\mathrm{d}w_{Y,t-1}) L_{Y,t-1} + (\mathrm{d}w_{H,t-1}) L_{H,t-1} + (\mathrm{d}R_t) V_{t-1} \Big) + R_1 \,\mathrm{d}V_0, \\
&= \partial_C U(C_1, L_1) \cdot \sum_{t=1}^{\infty} \frac{\beta^t \partial_C U(C_t, L_t)}{\partial_C U(C_1, L_1)} \Big((\mathrm{d}w_{Y,t-1}) L_{Y,t-1} + (\mathrm{d}w_{H,t-1}) L_{H,t-1} + (\mathrm{d}R_t) V_{t-1} \Big) + R_1 \,\mathrm{d}V_0, \\
&= \partial_C U(C_1, L_1) \cdot \sum_{t=1}^{\infty} R_{0\to t}^{-1} \Big((\mathrm{d}w_{Y,t-1}) L_{Y,t-1} + (\mathrm{d}w_{H,t-1}) L_{H,t-1} + (\mathrm{d}R_t) V_{t-1} \Big) + R_1 \,\mathrm{d}V_0.
\end{aligned}$$

In the case where $\gamma = 0$ (small open economy), we have that $R_t = \beta^{-1}$ which means that $\mathrm{d}R_t = 0$. □

C.2 Calibration details

We first extend the concept of "long-run" to a balanced growth path (BGP). Denote $\pi_X \equiv \log(X_{t+1}/X_t)$ the growth rate associated to a variable X. Differentiating the production functions for output and investment gives:

$$\begin{aligned}
\pi_Y &= \pi_{Z_Y} + \alpha \pi_K + (1-\alpha)\pi_{L_Y}, \\
\pi_H &= \pi_{Z_H} + \theta \pi_K + (1-\theta)\chi \pi_I + (1-\theta)(1-\chi)\pi_{L_H},
\end{aligned}$$

To obtain a balanced growth path with constant labor inputs, we assume that the productivity of production labor and investment labor grow at the same rate π. Expressed in terms of Hicks-neutral productivity, this is equivalent to assuming $\pi_{Z_Y} = (1-\alpha)\pi$, $\pi_{Z_H} = (1-\theta)(1-\chi)\pi$. In this case, all variables (K, Y, C, I, w_Y, w_H) grow at the same rate π. Using the optimality conditions, we obtain

$$\begin{aligned}
R &= \beta^{-1}(1+\pi)^{\gamma} \\
\frac{H}{K} &= \delta + \pi \\
q &= \frac{1}{r + \delta - \theta(\delta + \pi)} \frac{\alpha Y}{K}
\end{aligned}$$

The log return is:

$$\log R = -\log \beta + \gamma \pi$$

To calibrate β, we can use log returns minus $\gamma\pi$.

Second, combining (firm foc I_t) and (firm foc $L_{H,t}$), the long-run total investment yield is

$$\begin{aligned}
I + w_H L_H &= (1-\theta) q H, \\
&= (1-\theta) V \frac{H}{K}, \\
&= (1-\theta) V (\delta + \pi).
\end{aligned}$$

The total investment yield is therefore $\frac{I+w_H L_H}{V} = (1-\theta)(\delta+\pi)$: dilution is higher in a BGP with positive growth than in a steady-state. This is because the capital formation rate H/K is optimally higher.

Third, the capital share of income is

$$\begin{aligned}
\frac{\Pi}{Y+w_H L_H} &= \frac{\alpha Y}{Y+(1-\chi)(1-\theta)qH}, \\
&= \frac{\alpha}{1+(1-\chi)(1-\theta)\frac{qH}{Y}} \\
&= \frac{\alpha}{1+(1-\chi)\alpha\frac{(1-\theta)\delta}{r+(1-\theta)\delta}}
\end{aligned}$$

which is the same in both a steady-state and a BGP with positive growth. The last equation uses the fact that $qH = \frac{H/K}{r+(1-\theta)\delta}\alpha Y \implies q\frac{H}{Y} = \frac{\delta}{r+(1-\theta)\delta}\alpha$.

The last moment is the short-run, partial equilibrium tax elasticity. Consider a capital income tax shock $\mathrm{d}\tau_{K,t} = (1-\phi)^t \mathrm{d}\tau_K$ at $t=0$ around the undistorted steady-state. Using (firm foc I_t) and (firm foc $L_{H,t}$):

$$I_t + w_{H,t}L_{H,t} = (1-\theta)q_t H_t.$$

The log total derivative at $t=0$ is

$$\begin{aligned}
\mathrm{d}\log(I_0+w_{H,0}L_{H,0}) &= \mathrm{d}\log q_0 + \mathrm{d}\log H_0 \\
&= \left(1+\frac{1-\theta}{\theta}\right)\mathrm{d}\log q_0,
\end{aligned}$$

where the second equality is obtained using the fact that, in particular equilibrium, wages are taken as given. In that case, we obtain

$$\mathrm{d}\log(I_0+w_{H,0}L_{H,0}) = \frac{1}{\theta}\mathrm{d}\log q_0,$$

Using the envelope theorem on the firm value function, and using the fact that $\mathrm{d}w_{Y,t} = \mathrm{d}w_{H,t} = 0$, we obtain

$$\mathrm{d}\log q_0 = -\frac{r}{r+\phi}\frac{r+(1-\theta)\delta}{r}\mathrm{d}\tau_K$$

The first term accounts for the duration of the tax change, where ϕ represents an annual rate of decay. The second term accounts for the ratio of the tax base (capital income) to the capital payout. Putting together, we obtain

$$\mathrm{d}\log(I_0+w_{H,0}L_{H,0}) = -\frac{r+(1-\theta)\delta}{r+\phi}\frac{1}{\theta}\mathrm{d}\tau_K$$